

## 7. Black body again

a) Derive the Maxwell relations

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V \quad \text{and} \quad \left(\frac{\partial U}{\partial V}\right)_T = T \left(\frac{\partial p}{\partial T}\right)_V - p. \quad (11)$$

b) From his electromagnetic theory Maxwell found that the pressure  $p$  from an isotropic radiation field is equal to 1/3 the energy density  $u(T)$ :

$$p = \frac{1}{3}u(T) = \frac{U(T)}{3V}, \quad (12)$$

where  $V$  is the volume of the cavity. Show that  $u$  obeys the equation

$$u = \frac{1}{3}T \frac{du}{dT} - \frac{1}{3}u. \quad (13)$$

c) Solve this equation and obtain Stefan's law relating  $u$  and  $T$ .

## 8. Thermodynamic potential of an ideal gas

a) "Google" the notion "Guggenheim-Quadrat" in connection with thermodynamics, internalize the saying and draw the implied Quadrat. Which natural variables do the thermodynamic potential  $U$  (internal energy) and  $F$  (free energy) depend on?

b) So far, the thermal and caloric equation of state for the ideal gas are known:

$$pV = Nk_B T, \quad U = C_V(T - T_0) + U_0, \quad C_V = \text{const.} \quad (14)$$

Calculate the thermodynamic potential  $U$  for ideal gas with the help of Gibb's fundamental equation. Final result:

$$U = C_V T_0 \left[ \left(\frac{V}{V_0}\right)^{-\frac{Nk_B}{C_V}} \exp\left(\frac{S - S_0}{C_V}\right) - 1 \right] + U_0. \quad (15)$$

c) Also calculate the thermodynamic potential  $F$  for ideal gas.

## 9. Third law of thermodynamics

a) What does the third law of thermodynamics say?

b) Check whether the thermal equation of state

$$M = \frac{C}{T}H, \quad (16)$$

combining the magnetization  $M$  and the magnetic field  $H$  via the constant  $C$ , is compatible with the third law of thermodynamics.