4. Thermal and caloric equation of states again

In a pVT-system, the internal energy has the form

\[ U = U(V, T) = \alpha T^n \ln \left( \frac{V}{V_0} \right) + f(T), \]  

(7)

where \( f(T) \) is an arbitrary function of \( T \). Determine the constants \( \alpha \) and \( n \) under the assumption that the thermal equation of state can be written as

\[ p = T^3 V^{-1}. \]  

(8)

5. Irreversible Gas expansion

An ideal gas expands irreversibly and adiabatically from a volume \( V \) into vacuum \( \Delta V \).

a) What are the thermal and caloric equations of state for the ideal gas?

b) Starting from these equations, show that the entropy change of the process is described by

\[ \Delta S = N k_B \ln \frac{V + \Delta V}{V}. \]  

(9)

6. Change of entropy

Show that for the heat balance between two closed systems with particle number \( N_1, N_2 \) and the temperature \( T_1, T_2 \), an entropy change appears as

\[ \Delta S = C_V \ln \left[ \left( n_1 + n_2 \frac{T_2}{T_1} \right)^{n_1} \left( n_2 + n_1 \frac{T_1}{T_2} \right)^{n_2} \right], \]  

(10)

where \( n_i = \frac{N_i}{N_1 + N_2}, \) \( i \in \{1, 2\} \).

Which sign does \( \Delta S \) have? Why?