Department Physik
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Excercise to lecture

Theoretical Quantum Optics

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SHEET 9

1. Wigner function of a Fock state

For the Fock state $|n\rangle$, show that the Wigner function takes the form

$$W(\alpha) = (-1)^n \frac{2}{\pi} e^{-2|\alpha|^2} L_n(4|\alpha|^2)\,. \tag{1}$$

Plot eq. (4) for the Fock states with $n = 1$, $n = 2$, $n = 5$, and $n = 10$. Why is the Wigner function important in quantum optics?

Hints:

(1) Use the following definition for the Wigner function:

$$W(\alpha) = \frac{1}{\pi^2} \int d^2\beta\, e^{\alpha\beta^* - \alpha^*\beta} C^{(s)}(\beta)\,. \tag{2}$$

Here, $C^{(s)}(\beta) = \mathrm{Tr}\left(e^{\beta\hat{a}^\dagger - \beta^*\hat{a}}\rho\right)$ is the expectation value of the symmetrized displacement operator (why is that so?) and ρ is the density operator.

(2) Remember the disentanglement theorem!

(3) Of course, use the Fock basis.

(4) Use the identity

$$\frac{1}{\pi} \int d^2\alpha f(\alpha) e^{\alpha^* y - z|\alpha|^2} = z^{-1} f(z^{-1}y)\,. \tag{3}$$

2. Wigner function of the coherent state

Please verify, that the Wigner distribution for a coherent state takes the following form:

$$W(\alpha) = \frac{2}{\pi} e^{-2|\alpha-\beta|^2}\,. \tag{4}$$

The general Wigner distribution can be defined as

$$W(\alpha) = \frac{1}{\pi^2} \int d^2\lambda e^{\lambda^*\alpha - \lambda\alpha^*} C_N(\lambda) e^{-|\lambda|^2}\,. \tag{5}$$

$$C_N(\lambda) = <\beta| e^{\lambda\hat{a}^\dagger} e^{-\lambda^*\hat{a}} |\beta>\,. \tag{6}$$