Department Physik
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Paderborn, May 31$^{\text{th}}$ 2016

Excercise to lecture

Theoretical Quantum Optics

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SHEET 8

1. **Squeezed vacuum state**

   For the squeezed vacuum state, verify:

   (a) that

   $$C_{n+1} = -\frac{e^{i\theta}\sinh(r)}{\cosh(r)}\left(\frac{n}{n+1}\right)^{\frac{1}{2}} C_{n-1} \tag{1}$$

   is equivalent to

   $$C_{2m} = (-1)^m (e^{i\theta}\tanh(r))^m \left[\frac{(2m-1)!!}{(2m)!!}\right]^{\frac{1}{2}} C_0. \tag{2}$$

   (b) that $C_0 = \sqrt{\cosh(r)}^{-1}$.
   Hint: Use identities for the double factorial (double exclamation mark,see Wikipedia or something else for a definition) to simplify eq. (2).

   (c) that the probability of detecting $2m$ and $2m+1$ photons in the field is

   $$P_{2m} = \frac{(2m)!}{2^{2m}(m!)^2}\frac{(\tanh(r))^{2m}}{\cosh(r)} \quad \text{and} \tag{3}$$
   $$P_{2m+1} = 0\,. \tag{4}$$

2. **Thermal light**

   Given the thermal distribution

   $$P_n = \frac{\langle n\rangle^n}{(\langle n\rangle + 1)^{n+1}}\,, \tag{5}$$

   prove that

   $$(\Delta n)^2 = \langle n\rangle + \langle n\rangle^2 \tag{6}$$

   for thermal light. What follows from eq. (6)?

3. **Two-dimensional δ-function**

The two-dimensional δ-function occurs in normal order and antinormal order, eq. (7) and eq. (8), respectively:

$$\delta(\alpha^* - a^+)\delta(\alpha - a) = \frac{1}{\pi^2}\int \exp\left[-\beta(\alpha^* - a^+)\right]\exp\left[\beta^*(\alpha - a)\right] d^2\beta\,, \tag{7}$$

$$\delta(\alpha - a)\delta(\alpha^* - a^+) = \frac{1}{\pi^2}\int \exp\left[\beta^*(\alpha - a)\right]\exp\left[-\beta(\alpha^* - a^+)\right] d^2\beta\,. \tag{8}$$

Derive

(a) eq. (7) and

(b) eq. (8)

by starting on the RHS.

Hints:

(1) Take advantage of the effect of the annihilation operator on a coherent state $|\gamma\rangle$.

(2) Insert an appropriate "$\mathbb{1}$" at the appropriate position for eq. (8).

(3) Use the Fourier representation of the one-dimensional δ-function.