

Exercise to lecture  
**Theoretical Quantum Optics**

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SHEET 5

A coherent state, expanded in number states, is given by

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \quad (1)$$

**1. Coherent states**

Prove the following identities:

- (a)  $\hat{a}^\dagger |\alpha\rangle \langle \alpha| = (\partial_\alpha + \alpha^*) |\alpha\rangle \langle \alpha|$  and
- (b)  $|\alpha\rangle \langle \alpha| \hat{a} = (\partial_{\alpha^*} + \alpha) |\alpha\rangle \langle \alpha|$ .