Department Physik
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Excercise to lecture

Theoretical Quantum Optics

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SHEET 4

1. **A Fock state**

   Assume you have a Fock-state $|n\rangle$. Show that: $\hat{a}^{\dagger n}|0\rangle = \sqrt{n!}\,|n\rangle$.

2. **A coherent state**

   Assume you have a coherent state $|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle$. Show that:

   (a) $\langle\alpha|\alpha\rangle = 1$
   (b) $a|\alpha\rangle = \alpha|\alpha\rangle$.

3. **Baker-Hausdorff lemma**

   Proof:

   For any two operators $\hat{A}$ and $\hat{B}$, it is

$$e^{i\lambda\hat{A}}\hat{B}e^{-i\lambda\hat{A}} = \hat{B} + i\lambda\left[\hat{A},\hat{B}\right] + \frac{(i\lambda)^2}{2!}\left[\hat{A},\left[\hat{A},\hat{B}\right]\right] + ... \quad . \tag{1}$$

4. **Baker-Hausdorff-Campbell theorem**

   Proof:

   For $\left[\hat{A},\hat{B}\right] \neq 0$, but $\left[\hat{A},\left[\hat{A},\hat{B}\right]\right] = 0 = \left[\hat{B},\left[\hat{A},\hat{B}\right]\right]$, it is

$$e^{\hat{A}+\hat{B}} = \exp\left(\frac{1}{2}\left[\hat{A},\hat{B}\right]\right) e^{\hat{B}}e^{\hat{A}} = \exp\left(-\frac{1}{2}\left[\hat{A},\hat{B}\right]\right) e^{\hat{A}}e^{\hat{B}} \,. \tag{2}$$

5. **Variance**

   Show that the variance of an observable $\hat{A}$ is given by

$$\langle(\Delta\hat{A})^2\rangle = \langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2 \,. \tag{3}$$