Department Physik
Universität Paderborn

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Excercise to lecture

Theoretical Quantum Optics

Dr. Matthias Reichelt
Dr. Polina Sharapova

SHEET 3

6. **Mean energy**

An each field mode can be considered as a harmonic oscillator. According to the Boltzmann distribution the probability to measure energy E of harmonic oscillator is given by

$$w(E) = Aexp[-\frac{E}{k_b T}], \quad (1)$$

where A is the normalization constant, T is the temperature, k_b is the Boltzmann constant. Following the definition, the mean value of some variable (energy in our case) can be calculated as

$$< E >= \frac{\int Ew(E)dE}{\int w(E)dE}, \quad (2)$$

$$< E >= \frac{\sum_n E_n w(E_n)}{\sum_n w(E_n)} \quad (3)$$

for the continuous and discrete spectrum correspondingly. Calculate the mean energy value in classical case (each mode has energy E, continuous spectrum) and in the quantum case: $E_n = n\hbar\omega$ - each mode has its own energy, discrete spectrum. Plot the spectral density

$$\rho = \frac{\omega^2}{\pi^2 c^3} < E > \quad (4)$$

in each case.

7. **Probability distributions**

The Poissonian probability distribution P_p is given by

$$P_p(X = k) \quad = \quad \frac{\lambda^k}{k!} \cdot e^{-\lambda} \text{ with } n \gg 1,\, p \ll 1 \text{ and } \lambda = np\,. \quad (5)$$

The analysis of reports of car accident during 5 years in a town is described in the table below. Compare the table with theoretical values given by the poisson distribution.

number of deaths per accident	0	1	2	3	4
number of accidents	109	65	22	3	1

Table 1: Analysis of reports of car accidents during 5 years in an arbitrary town.