Department of Physics
University of Paderborn

Paderborn, 26th Apr. 2016

Exercise

Computational Optoelectronics and Photonics

Dr. M. Reichelt WS 2016

PROBLEM SHEET III

Please prepare by next exercise.

4. **The Workhorse**[1]

Solve the differential equation (1) of PROBLEM SHEET II using the **4th-order-Runge-Kutta** method.

- Write the algorithm in a general form which can handle an arbitrary number *N_eqn* of coupled first order differential equations, i.e., all variables should be of the form

  ```
  complex parr[N_eqn];
  ```

  These enter the Runge-Kutta function

  ```
  void rk4(void);
  ```

  and the "right-hand-side" function

  ```
  void derivs(complex y[], complex dydx[]);
  ```

- Prove that the program gives the correct result by setting $N\_eqn = 2$ and reproducing the oscillation obtained via the *leap-frog* algorithm.
  Use the solution of problem 3(c) to find the eigenfrequency $\omega = 2\pi/T$.
- The equations of motion (using the small angle approximation) for two coupled pendulums are

$$
\begin{aligned}
\ddot{\phi}_1 &= -\frac{g}{l}\phi_1 - \frac{k}{m}(\phi_1 - \phi 2) \\
\ddot{\phi}_2 &= -\frac{g}{l}\phi_2 + \frac{k}{m}(\phi_1 - \phi 2).
\end{aligned} \tag{3}
$$

  Calculate the two new eigenfrequencies by applying the appropriate initial conditions. Use $g/l = 9$ and $k/m = 8$. (What are the analytic results for $\omega_{1,2}$?)
- Now calculate 10000 pendulums at the same time. If you define the arrays within a function the program will crash. (The reason is that local variables are put on the stack which has a limited size.) What are the three ways to overcome this problem?

[1] Quote from the Numerical Recipes chapter 17.1.