Department of Physics
University of Paderborn

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Exercise

Computational Optoelectronics and Photonics

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PROBLEM SHEET II
Please prepare by next exercise.

3. **Simple Pendulum**
The equation of motion for a pendulum is given by

$$\ddot{\phi} = -\frac{g}{l}\sin(\phi)\,. \tag{1}$$

(a) Rewrite Eq. (1) as a system of two first order ordinary differential equations and write a program to solve these coupled equations with the help of the *leap-frog* algorithm.

(b) Plot the solution $\phi(t)$ for several periods and different initial conditions with GNUPLOT or/and MATLAB. Choose $\omega(t = 0) = 0$ (pendulum at rest at the beginning) and different initial angles $\phi(t = 0)$.

(c) Change your code in such a way as to determine the length of *one period*. Then, find out numerically up to which starting angle the small angle approximation ($\sin\phi \approx \phi$) holds. Let's say the deviation from the exact solution should be less the 1%. What would you expect from the formula given in the lecture

$$T \approx 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{16}\phi_{\max}^2\right) \quad ? \tag{2}$$