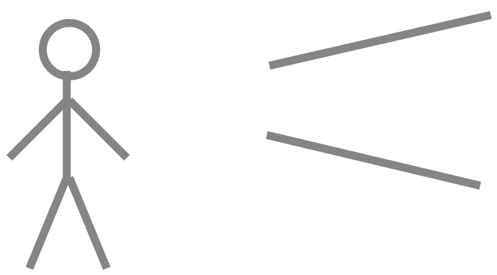
Pseudonyms in Cost Sharing

Paolo Penna • Riccardo Silvestri • Peter Widmayer • Florian Schoppmann

Pseudonyms

"What makes mechanisms
immune to fake identities?"



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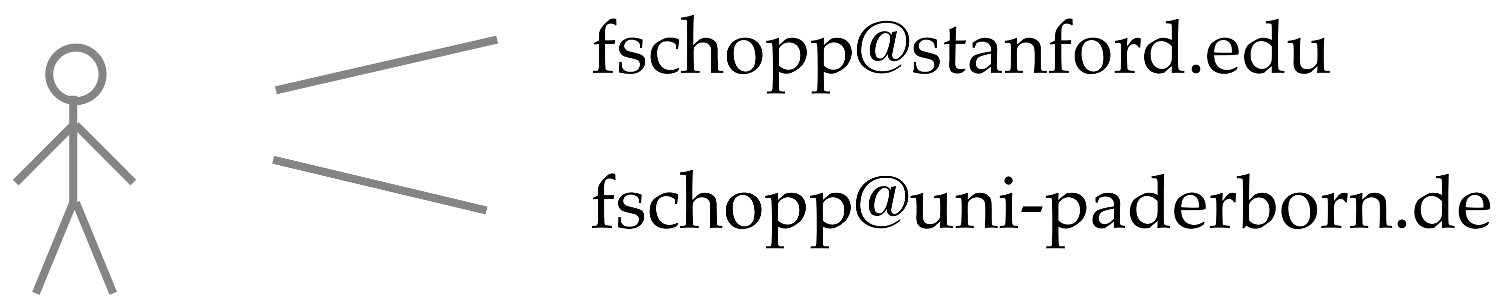

- Virtual identities are cheap

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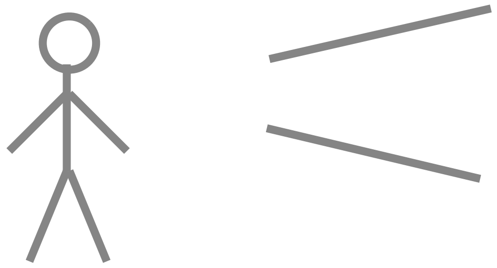

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- Virtual identities are cheap
- Similar in spirit to falsename-proofness
  (Yokoo et al., GEB'04)

Cost Sharing

"**Who** should participate in a joint project and
at what price?"

Automated Negotiations in logistics

Infrastructure for broadband internet access

Car Sharing

Cost-Sharing Mechanisms

- Who should participate and at what price?

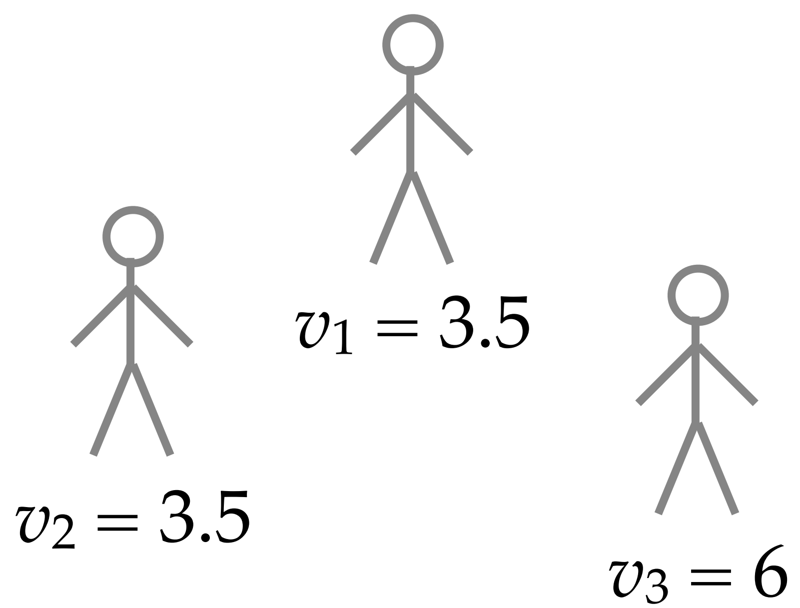
$\boldsymbol{b}$ →  → $Q \subseteq \{1, \ldots, n\}$

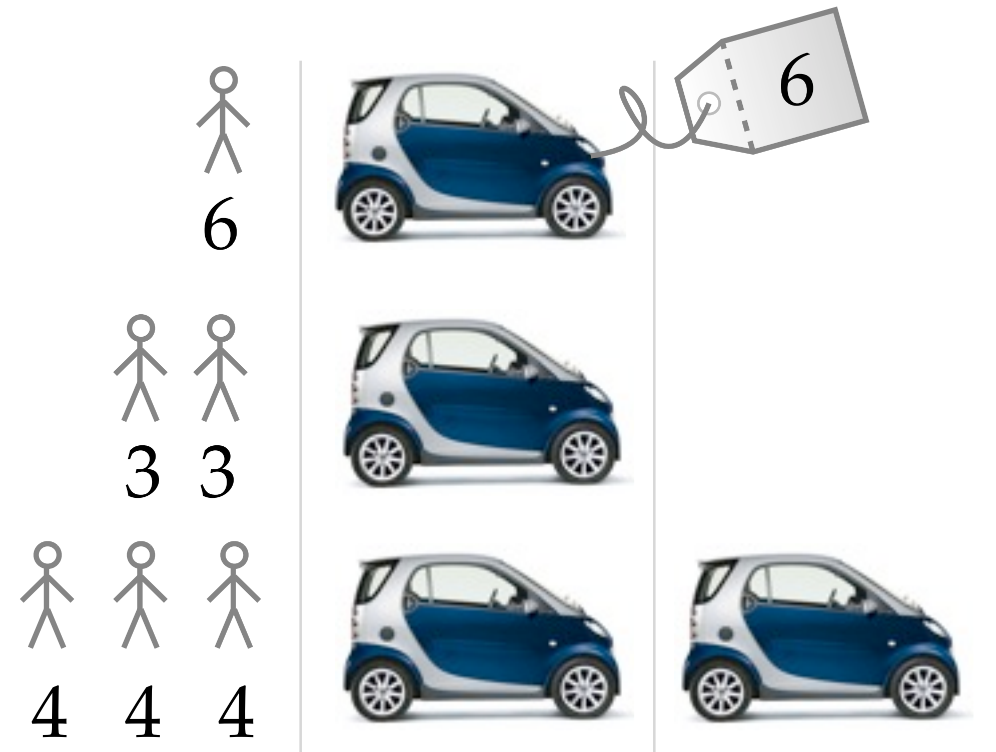
→ $\boldsymbol{x} = (x_1, \ldots, x_n)$ where $x_i \in [0, b_\mathrm{i}]$

- Typical requirements
  - Approximate budget balance: $C(Q) \leq \sum x_i \leq \beta \cdot C(Q)$
  - Economic efficiency (relaxed here: consumer sovereignty)
  - Strategy-proofness, strategic players optimize net utility $= \begin{cases} v_i - x_i & \text{if } i \in Q \\ 0 & \text{else} \end{cases}$

Example: Car Sharing

Identical prices, iteratively drop all underbidders?





- For non-submodular costs: $C(Q) = 6 \cdot \left\lceil \frac{|Q|}{2} \right\rceil$
  - SP and BB mutually exclusive with identical prices

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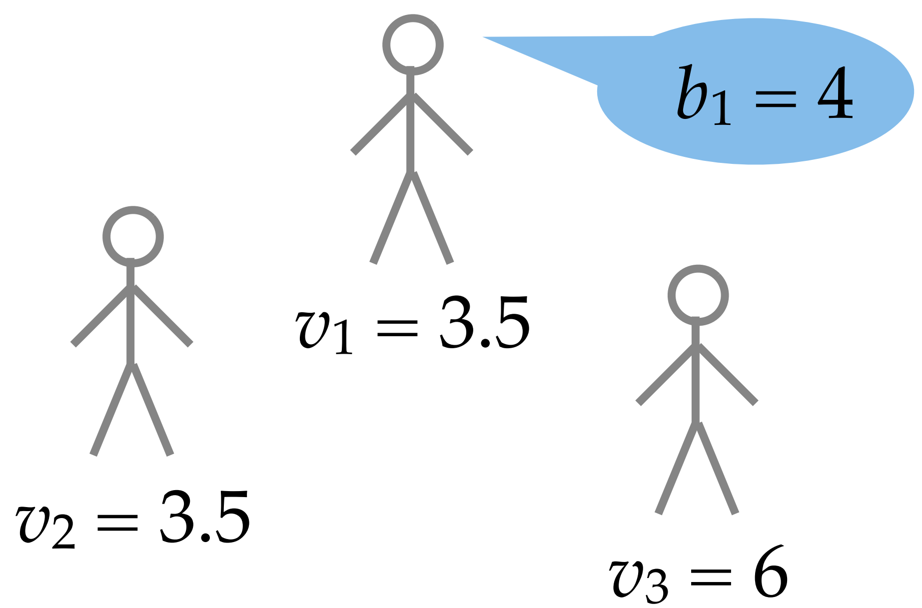

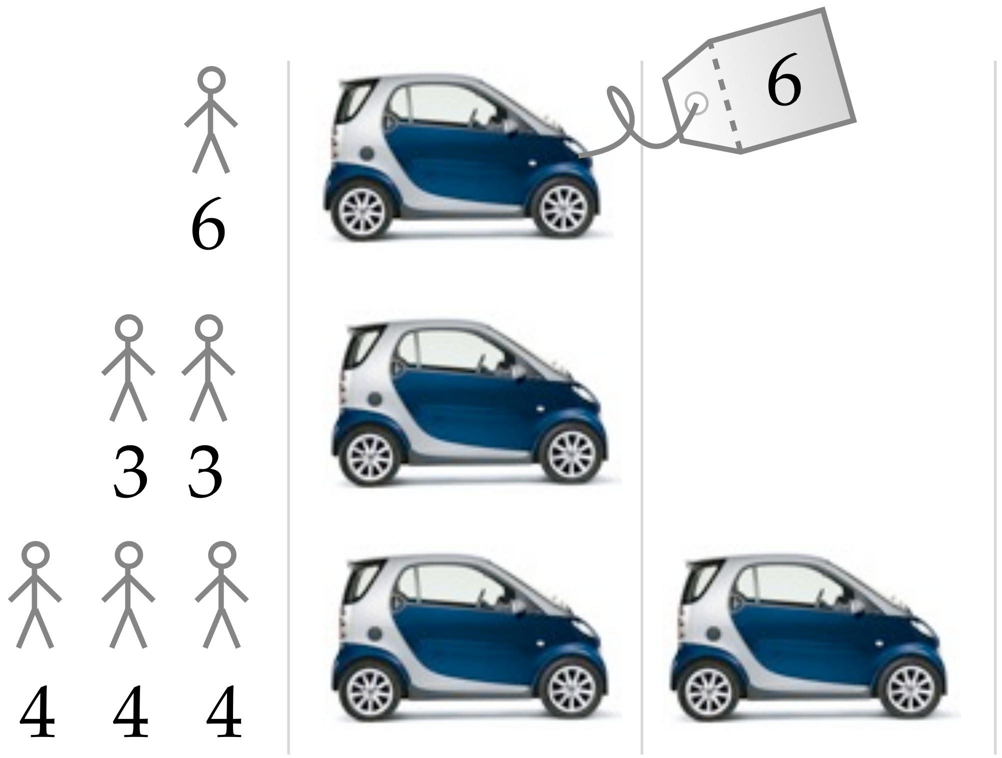

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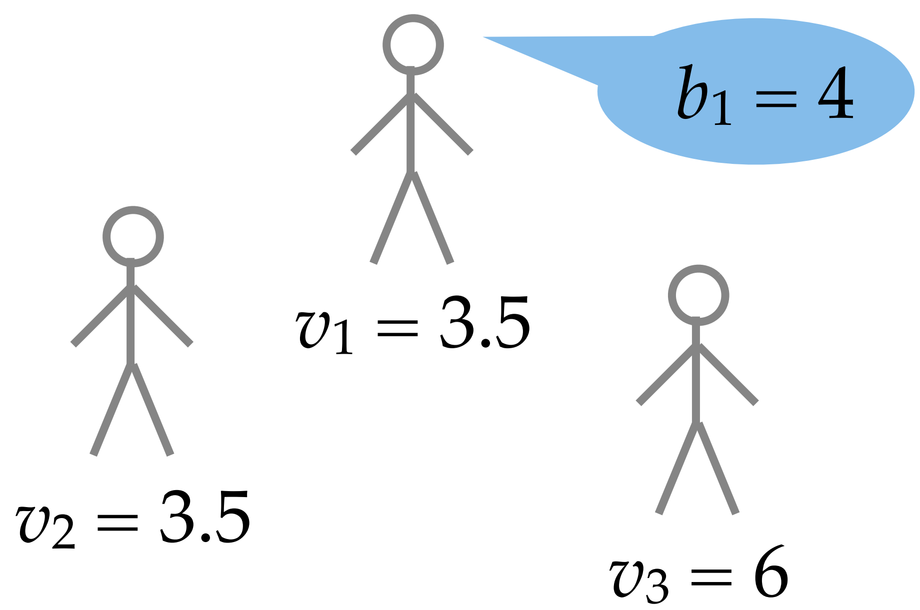

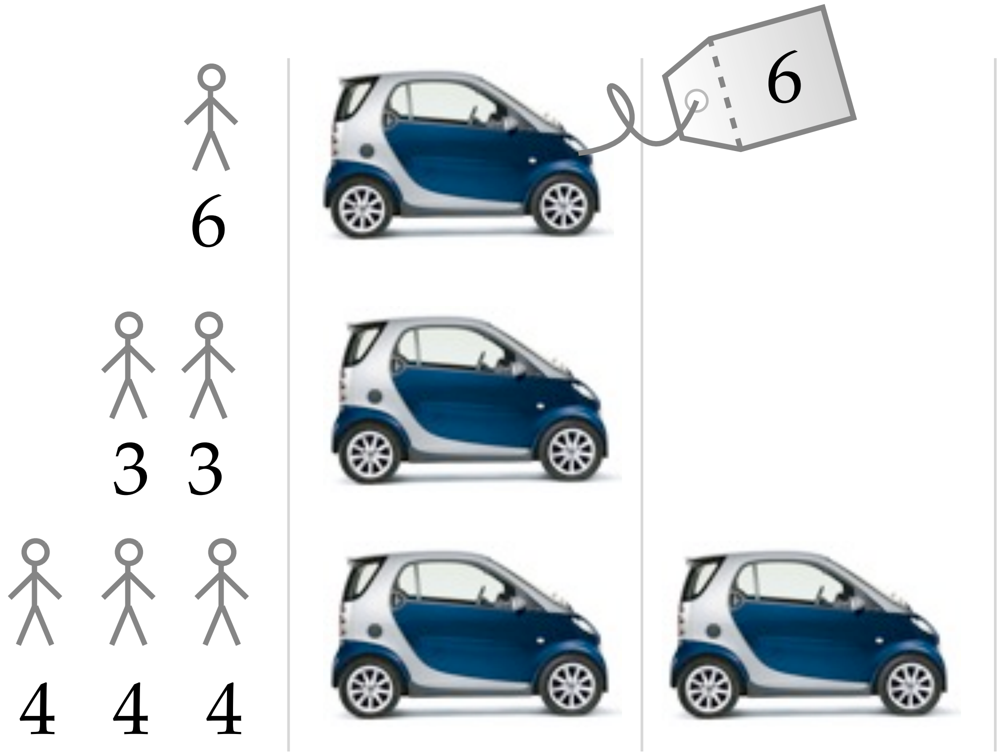

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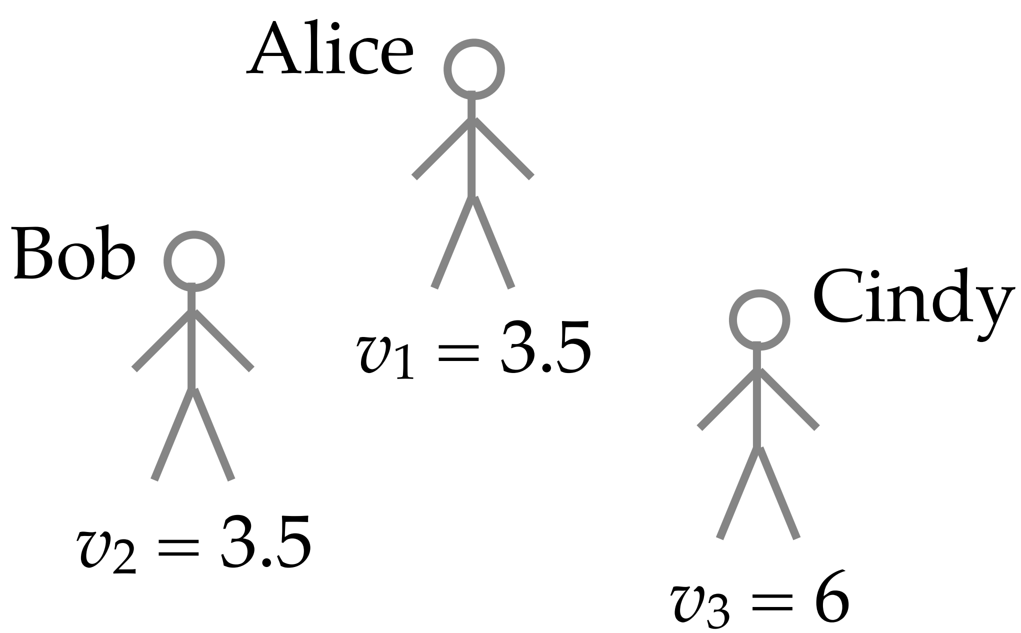
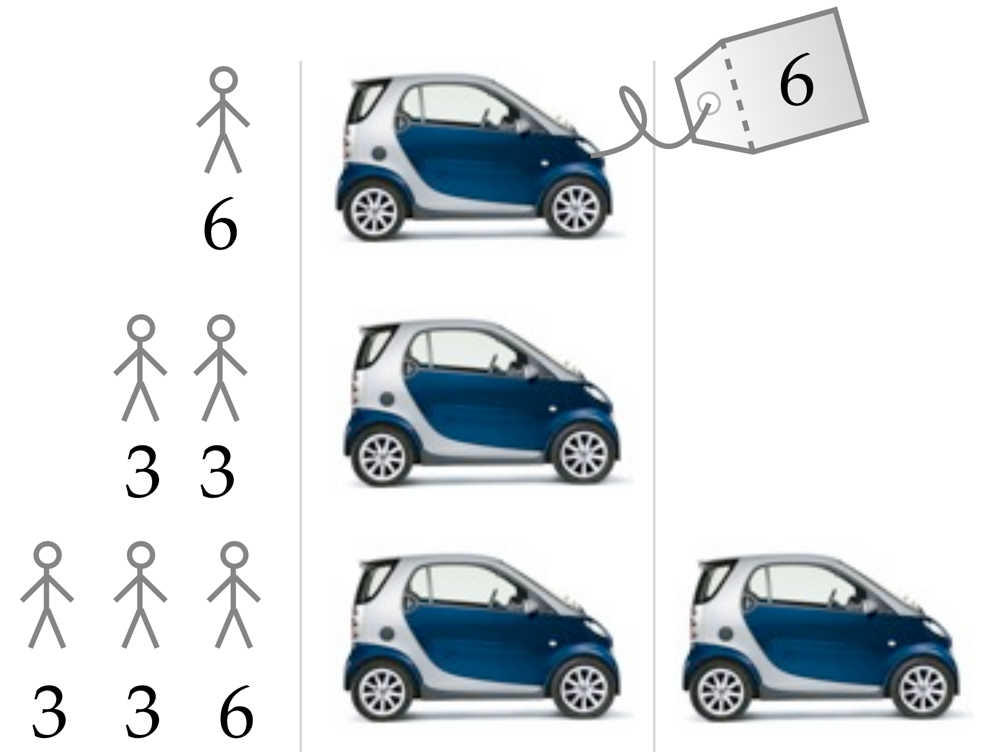
Example: Car Sharing

Serve first two players i bidding $b_i \geq 3$ for price 3, all others for price 6





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  - No multiple bids

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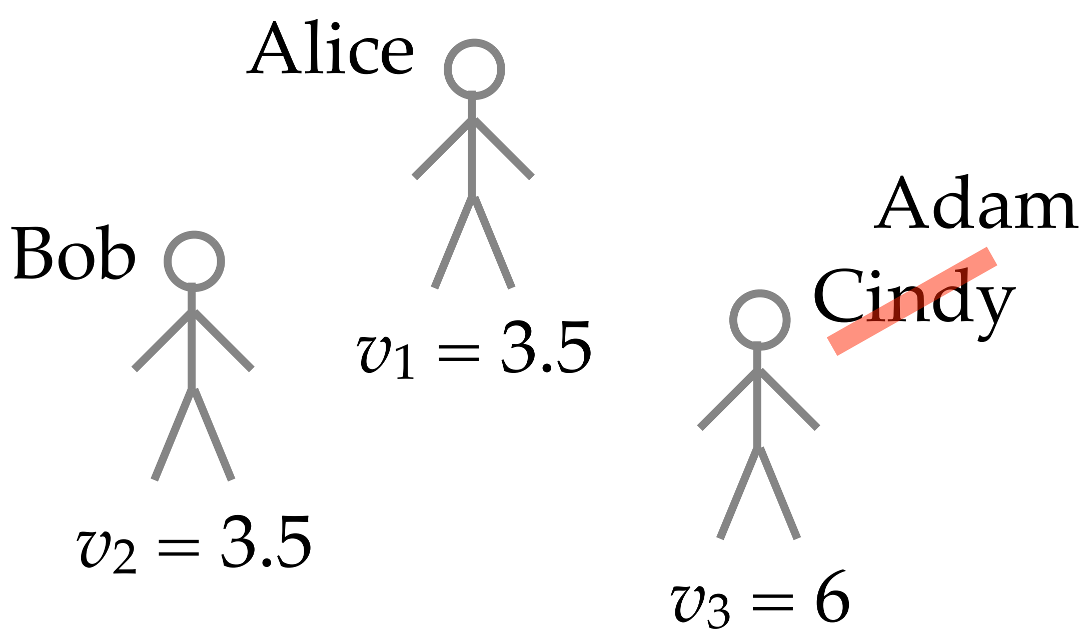

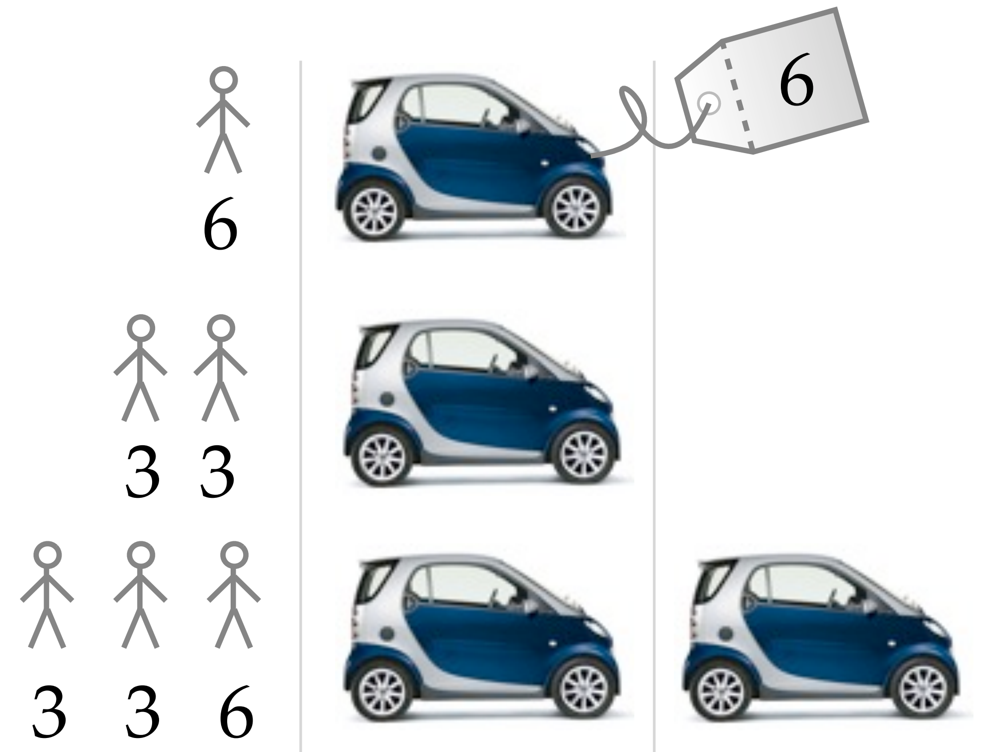

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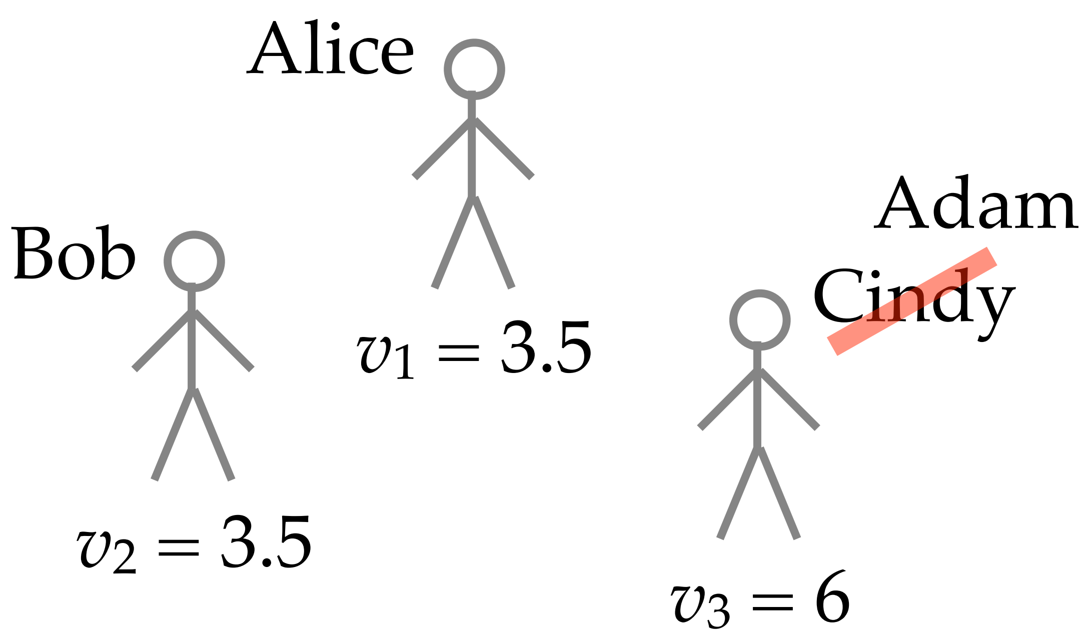

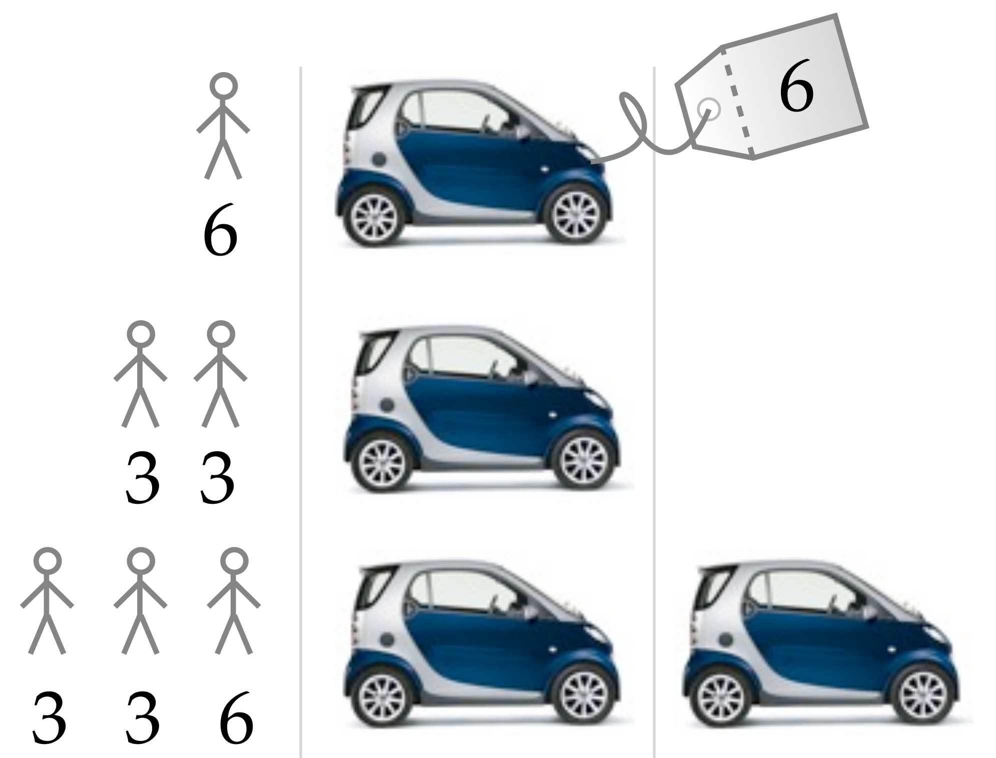

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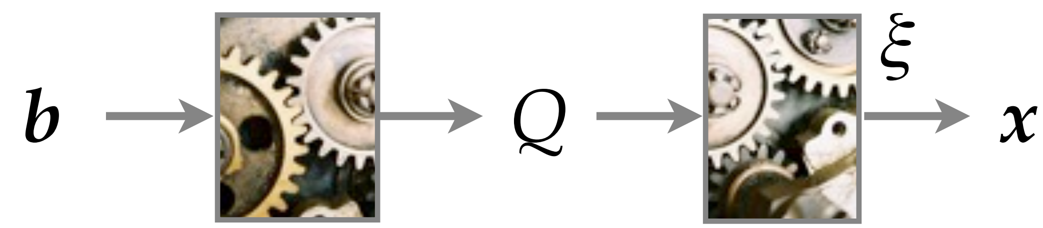
Randomization?

- Non-trivial if also collusion resistance required
  - Randomization over collusion-resistant mechanisms $\not\Rightarrow$ collusion-resistant in expectation
  - And vice versa (Goldberg, Hartline, SODA'05)
- not very common in cost-sharing literature
  - should only be means, but not an end

Previous Techniques

- Separability: $\boldsymbol{x} = \xi(Q)$



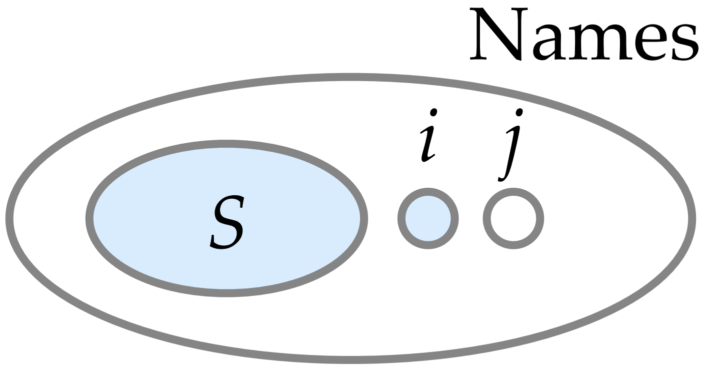
- Moulin mechanisms (Moulin, Soc Choice Welf'99)
  - Cross-monotonic cost shares: $\xi_i(S \cup j) \leq \xi_i(S)$
  - Choose largest $\boldsymbol{b}$-feasible set, i.e., $\forall i \in Q$: $b_i \geq \xi_i(Q)$

$Q := \{1, \ldots, n\}$
while $\exists\, i$: $b_i < \xi_i(Q)$
  $Q := Q \setminus i$

- Acyclic mechanisms (Mehta et al., EC'07)
- Two-price mechanisms (Bleischwitz et al., MFCS'07/09)

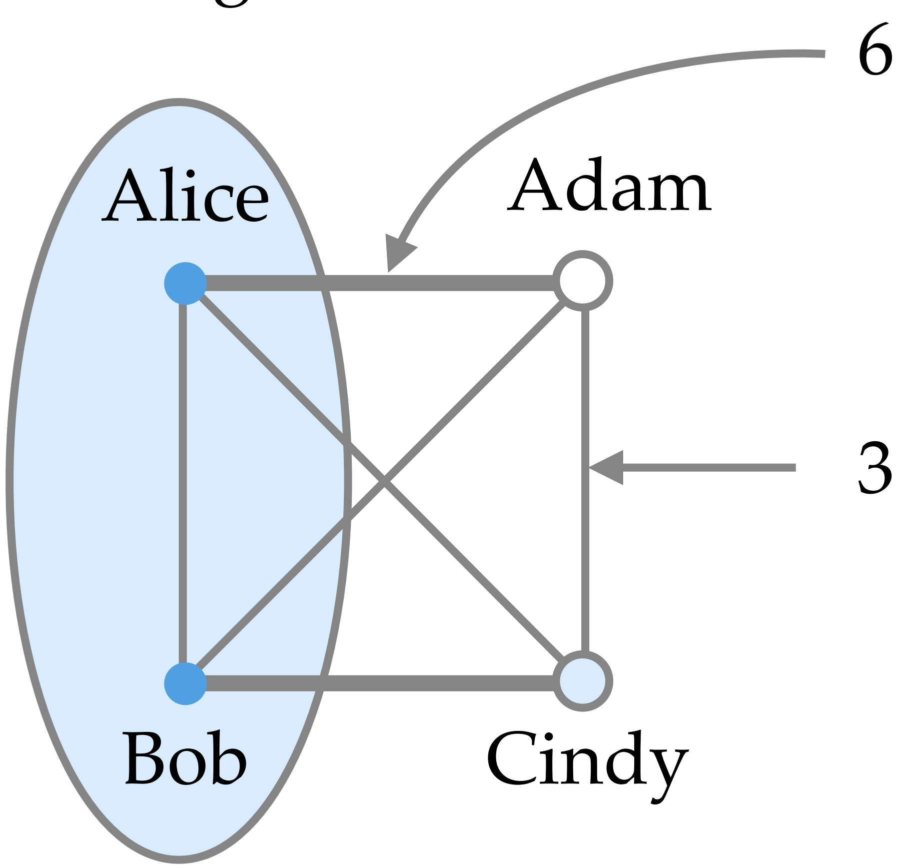
Name independence

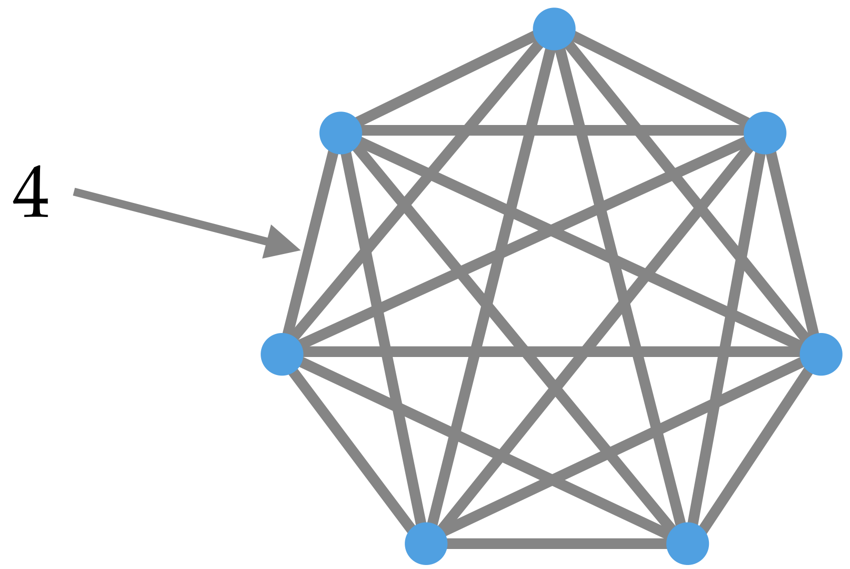
- A (separable) reputationproof cost-sharing mechanism satisfies $\xi_i(S \cup i) = \xi_j(S \cup j)$ for all S and $i,j \notin S$
  - Follows from consumer sovereignty



Example

- Serving three players
  - Assign a weight to all edges
  - Exact budget balance: For all triangles, sum of edge weights must be 12





Hypergraphs

Hypergraphs

- Cost shares for s-player sets:
  - Consider complete $(s-1)$-uniform hypergraph
  - Assign weight to each hyperedge so that for all s-subsets the sum of all its hyperedges' weights is $\in [1, \beta]$

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- System of linear inequalities:

$$\overbrace{(1, \ldots, 1) \leq A \cdot \boldsymbol{x} \leq (\beta, \ldots, \beta)}$$

$$= \Big([R \subset S]\Big)_{S \text{ is } s\text{-set}, R \text{ is } (s-1)\text{-set}} \in \{0,1\}^{\binom{n}{s} \times \binom{n}{s-1}}$$

  - Gottlieb (Proc. of AMS'66): Incidence matrix A has full rank

Hypergraphs

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$$Poly := \{\, \boldsymbol{x} \mid (1, \ldots, 1) \leq \overbrace{A}^{= \left([R \subset S]\right)_{S \text{ is } s\text{-set}, R \text{ is } (s-1)\text{-set}} \in \{0,1\}^{\binom{n}{s} \times \binom{n}{s-1}}} \cdot \boldsymbol{x} \leq (\beta, \ldots, \beta) \,\}$$

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How much can cost shares differ?

- Suppose $\boldsymbol{x}$ and Q are such that x_Q is minimal
  - W.l.o.g. assume $x_R = x_{R'}$ for all R, R' with $|Q \cap R| = |Q \cap R'|$
  - Let p_k unique value with $x_R = p_k$ for all R with $|Q \cap R| = k$
  - Then $x_Q = p_{s-1}$ and for evey s-subset S with $k = |S \cap Q|$

$$(A \cdot \boldsymbol{x})_S = \sum_{\substack{R \text{ is } (s-1)\text{-} \\ \text{subset}}} [R \subset S] \cdot x_R = k \cdot p_{k-1} + (s-k) \cdot p_k =: b_k \in [1, \beta]$$

  - Thus, $p_{s-1} = \sum_{i=0}^{s-1} \frac{(-1)^{s-i-1} \cdot b_i}{s-i} \cdot \binom{s-1}{s-i-1}$ monotone in every b_i
- With a short calculation:

$$x_Q = \min \{x_R \mid \boldsymbol{x} \in \textit{Poly}, R \text{ is } (s-1)\text{-subset}\} = \frac{2^{s-1}(1-\beta)+\beta}{s}$$

But...

- For any $\delta > 0$, given a large enough name space, for each cardinality s there is an s-set S with

$$\forall i, j \in S : |\xi_i(S) - \xi_j(S)| \leq \delta$$

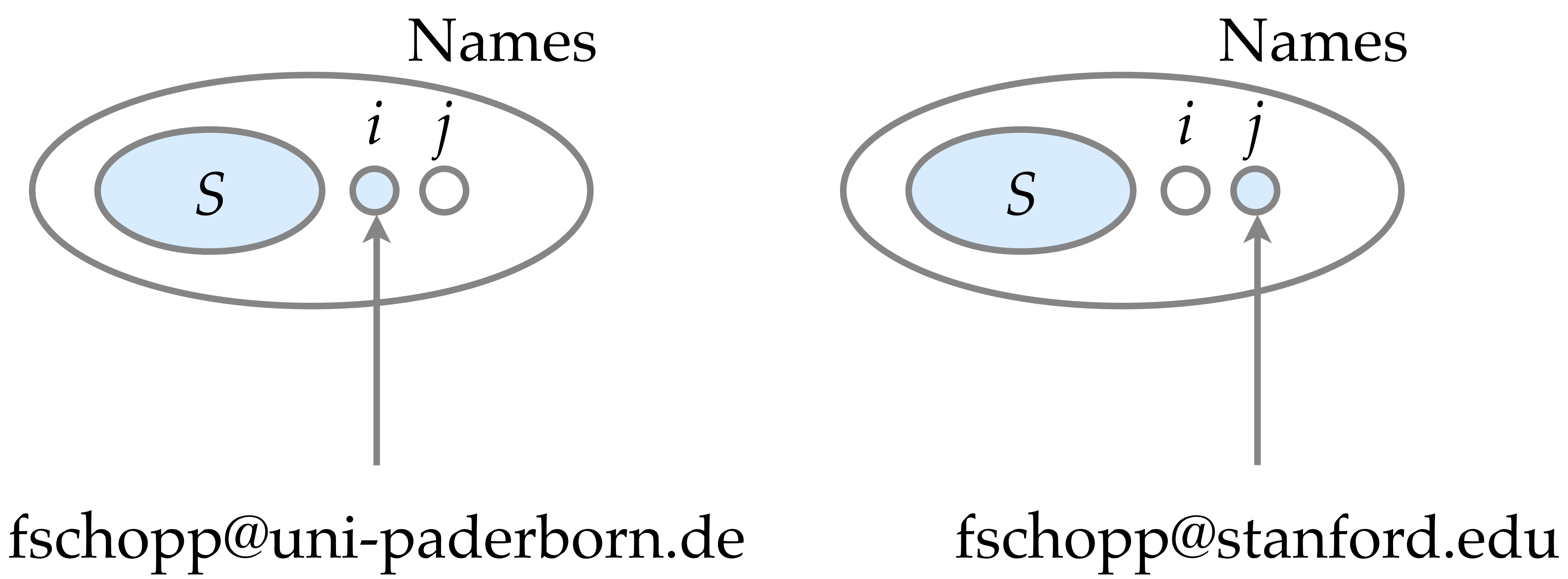
  - When finite number of prices: Coloring
  - Ramsey's Theorem (Proc. London Math. Soc'30):

    Let $c, r, s \in \mathbf{N}$ with $s \geq r$. Then $\exists\, n$: If the r-subsets of any n-set are colored with c colors: $\exists\, s$-set all of whose r-subsets have the same color.

Implications

- Characterizations of identical prices
  - New: Separable + 1-budget balance + reputationproof (when at most half the names in use)
  - For excludable public good (i.e., $C(Q) = 1 \Leftrightarrow Q$ nonempty) previous characterizations due to, e.g., Dobzinski et al. (SAGT'08) and Deb and Razzolini (Math. Soc. Sciences'99)
- Impossibility
  - Separable, strategyproof, reputationproof, and 1-budget balanced w.r.t. non-submodular costs

Ralax Rename-proofness



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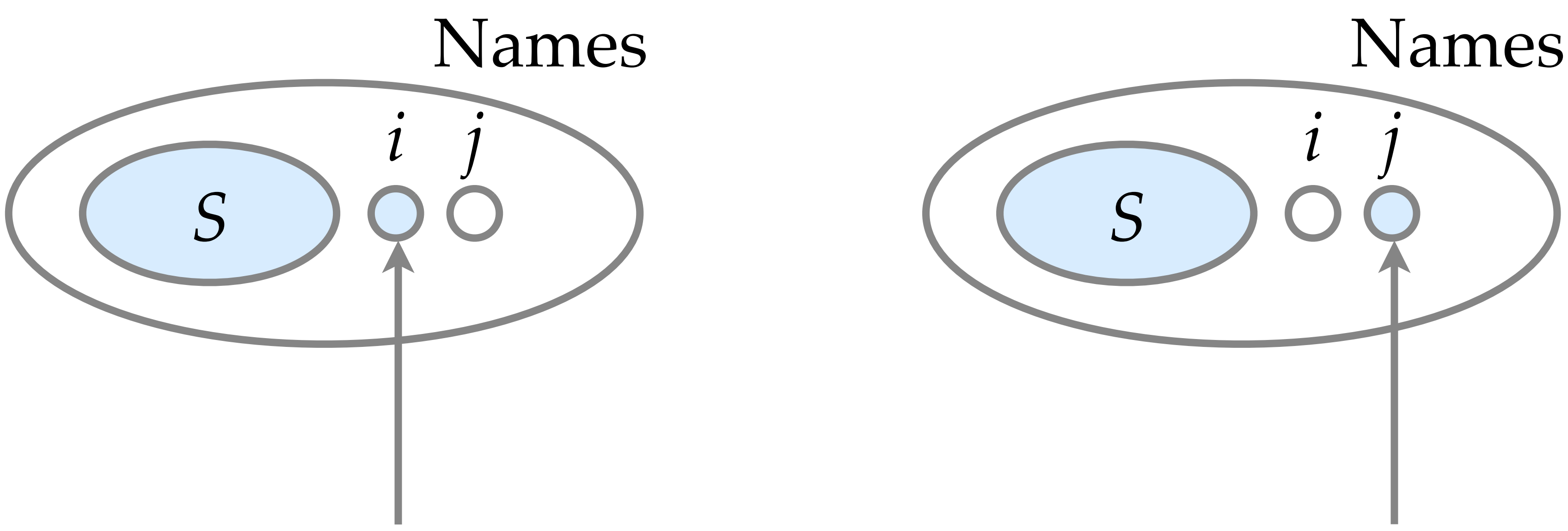

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1 year ago

Feedback: 107 positives

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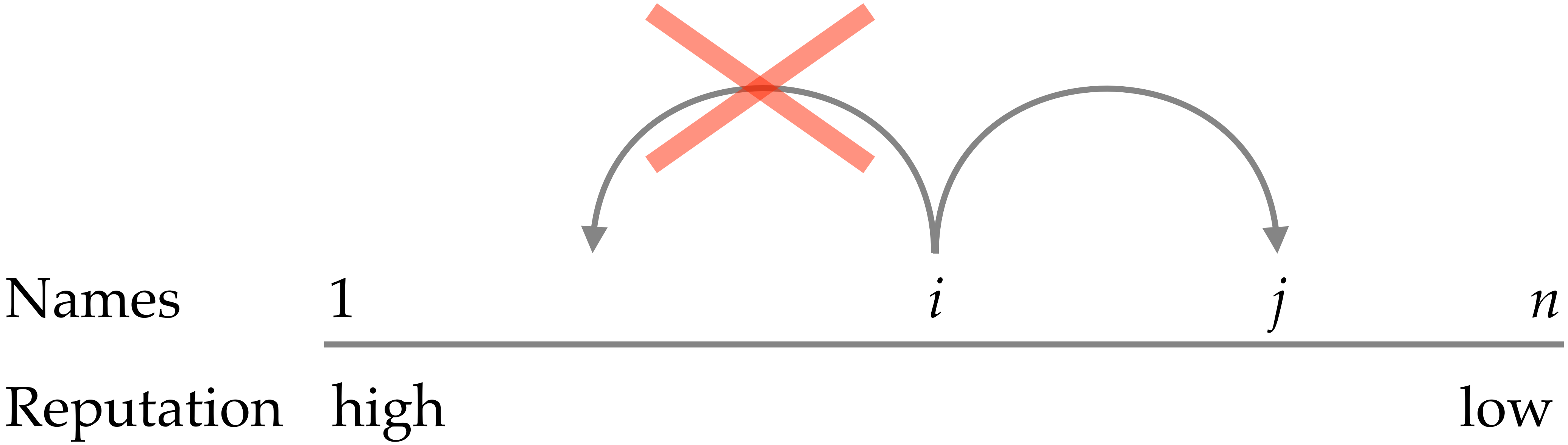
2 min ago

Feedback: 1 positive

- Use reputation for ranking players!

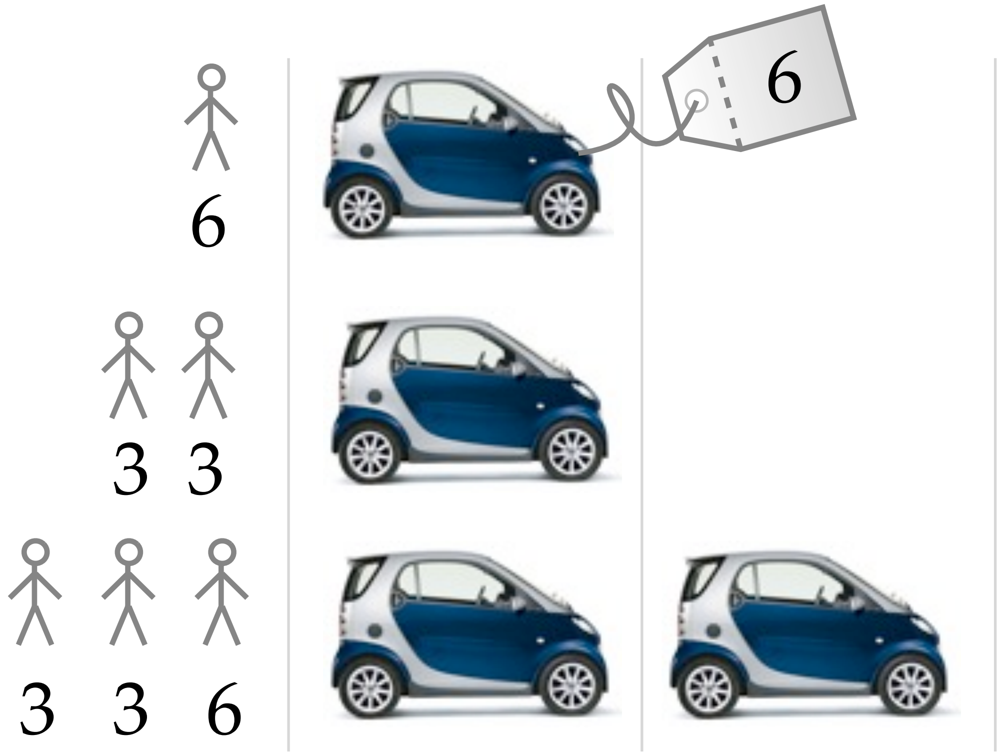
Reputationproof



- No player i can increase her utility unilateraly by bidding with a pseudonym $j > i$

Example

- Serve set Q that lexicographically maximizes the vector of net utilities
  - This mechanism is reputationproof
  - This mechanism is also group-strategyproof (Bleischwitz et al. MFCS'07/09)



$$C(Q) = 6 \cdot \left\lceil \frac{|Q|}{2} \right\rceil$$

Conclusion

- Renameproof
  - Identical prices or randomized mechanisms
- Reputationproof
  - better reputation $\Rightarrow$ better price
  - In some sense a reasonable derandomization
  - Most known mechanisms not reputationproof in general

Thanks!